Persistent current of two-chain Hubbard model with impurities

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Abstract

The interplay between impurities and interactions is studied in the gapless phase of two-chain Hubbard model in order to see how the screening of impurity potentials due to repulsive interactions in single-chain model will be changed by increasing the number of channels. Renormalization group calculations show that charge stiffness, and hence persistent current, of the two-chain model are less enhanced by interactions than single chain case.

I. INTRODUCTION

Interacting electrons in a disordered system constitue one of the most challenging problems. Questions on roles of interactions and disorder in quantum wires were again cast by the experiments of persistent current in mesoscopic rings [1]. The ground state of a ring system with a magnetic flux breaks time reversal symmerty and has a finite circulating current as a periodic function of the flux. This persistent current was predicted theoretically long time ago [2], but we had to wait a few decades until technological progress allowed to observe the current. The experiments revealed that the ring supports a large current even with modest amount of impurities. Non-interacting electrons with impurities are shown [3] to have persistent current with a factor l/L multiplied, where l is the elastic mean free path and L the ring size. The current observed in the experiments was much larger than this value and other effects had to be taken into account to explain the large current. Electron-electron interactions were considered to be the first candidate as an origin of the large current, because a simple consideration gives a scenario that repulsive interactions would prevent particles from gathering in deep impurity holes and hence the interacting electrons would not be easily pinned by impurities, compared to non-interacting electrons. However the problem is not that simple as shown in the studies of one-dimensional (1D) system. For example, the roles of interactions are quite different between the systems with spinning and spinless electrons. The detail of the difference is summarized in section II.

Since 1D systems allow us to study the physical properties rather rigorously with the help of bosonization and renormalization group (RG) techniques. Approaching the problem

of persistent current from a viewpoint of 1D systems, hence, has this kind of advantage. Analytical and numerical studies have been done for interacting electrons in a single chain with disorder potentials. Summary of their results is given also in section II.

In order to approach more realistic model, one need to consider multi-channel effects on the chain. Recent growing interests in few-coupled chains are mostly focused on pure systems and there have been only a few studies on ladder electron systems with impurity potentials. In this paper we study two-chain Hubbard model with impurities as a first step from a single chain towards a finite cross section ring. In section III we derive RG equations of the renormalized strengths of impurity potentials in the gapless phase, and show how the relation between interactions and disorder is changed by increasing the number of channels. Summary is given in section IV.

II. SINGLE CHAIN

Let us first begin by summarizing the impurity-interaction effects on persistent current of single chains. For spinless electron models with impurities, analytical and numerical studies showed that persistent current is suppressed by interparticle repulsive correlation as far as the impurity potentials are weak enough [4–6]. A RG calculation gives the effective backward impurity scattering W is renormalized as $\mathrm{d}W/\mathrm{d}l = (3 - 2K_\rho)W$ where K_ρ is the Luttinger parameter. [4] The larger repulsive interactions give rise to the smaller K_ρ and therefore to the stronger effective impurity potentials, which would diminish the persistent current. This is due to the enhancement of charge density wave (CDW) correlations in the ground state of the system. The repulsive interactions would enhance the CDW correlations, which make the system easily pinned by the impurities therefore make the persistent current smaller than the value of non-interacting systems.

In the presence of the strong impurity potentials, on the other hand, the wave functions are almost localized. When the interactions are turned on between the localized particles, they start to escape from each other and the localization of the particles in the deep impurity potentials is relaxed effectively. As a consequence, the interactions tend to enhance the persistent current of systems with strong impurity potentials [6].

In the experimental point of view, the above results are disappointing because the materials used in the experiments are all in the diffusive or ballistic regime so the scenario of strong impurity potentials cannot be applied.

The situation changes drastically, however, when the spin degrees of freedom are taken into account [7]. The renormalized impurity potential is given by $\mathrm{d}W/\mathrm{d}l = (3 - K_\rho - K_\sigma - y)W$, where K_ν $(\nu = \rho, \sigma)$ is the Luttinger parameters of charge and spin sectors respectively and y measures the backward scattering strength between electrons of opposite spins. For Hubbard model with small U, this RG equation becomes

$$\mathrm{d}W/\mathrm{d}l = (1 - U/\pi v_F)W, \tag{1}$$

where v_F is the Fermi velocity. Here electron-electron interactions would screen impurity potentials. Since spin density wave (SDW) correlation is dominant in the ground state of the repulsive Hubbard model, the interaction U makes the particle density uniform, and therefore make the coupling of the density to the impurities weak. This role of interactions

is a good news to explain the experimental data: electron-electron interactions weaken the impurity effects and the persistent current would be enhanced from the value of non-interacting systems.

Note that, while $2k_F$ component of density-impurity coupling is renormalized to smaller value due to the interactions as shown, $4k_F$ component grows in the presence of the interactions. Since k_F of spinning electrons is equal to $2k_F$ of spinless fermions, $4k_F$ coupling in infinite U Hubbard model corresponds to $2k_F$ coupling of spinless model where the $2k_F$ coupling is enhanced by the interactions as stated above. Therefore as U increases the effective strength of impurity potentials decreases as long as U is small and then increases for large U because of the effect of $4k_F$ component [8].

Those stories are about short range interactions. Then, what about long-range Coulomb interactions? Since it is shown [9] that long-range interactions make $4k_F$ density correlations dominant, the interaction would enhance the effective strengh of impurity potentials and hence suppress persistent current. The suppression of persistent current by Coulomb interactions was observed in a numerical calculation [10].

Those analyses have almost revealed the interplay between interactions and impurities in single chain systems. For more realistic discussions, however, one has to consider multi-channel effects because the real ring systems have finite cross sections. Although it is shown that the persistent current of single-chain Hubbard model would be enhanced by the presence of electron-electron interactions, it is not clear whether this effect still remains in multi-channel systems and, more precisely, whether this effect will become stronger or weaker as the number of channels increases. In order to see this, we study two-channel Hubbard model as a starting point toward multi-channel models.

III. TWO-CHAIN LADDER

In this section we study two-chain Hubbard model which is described by $H = H_0 + H_{int}$;

$$H_0 = -t \sum_{\alpha,s,i} c^\dagger_{\alpha s i} c_{\alpha s i+1} - t_\perp \sum_{s,i} c^\dagger_{1 s i} c_{2 s i} + \text{h.c.}, \tag{2}$$

$$H_{int} = U \sum_{\alpha i} n_{\alpha \uparrow i} n_{\alpha \downarrow i}, \tag{3}$$

where α $(= 1,2)$ is chain index. H_0 can be diagonalized by $a_{o,\pi} = (c_1 \pm c_2)/\sqrt{2}$, where o and π denote the bonding and anti-bonding bands respectively. Using the standard bosonization scheme for fermions with spins, we can describe the system by four fields $\phi^\pi_\rho, \phi^\pi_\sigma, \phi^o_\rho, \phi^o_\sigma$. Then, introducing a linear combination of the fields: $\phi_{\nu\pm} = (\phi^o_\nu \pm \phi^\pi_\nu)/\sqrt{2}$ where $\nu = \rho, \sigma$, we have the Hamiltonian, $H = H_1 + H_2 + H_3$;

$$H_1 = \sum_{\substack{\nu = \rho, \sigma \\ r = \pm}} \int \frac{dx}{2\pi} [u_{\nu r} K_{\nu r} (\pi \Pi_{\nu r})^2 + \frac{u_{\nu r}}{K_{\nu r}} (\partial_x \phi_{\nu r})^2], \tag{4}$$

$$H_2 = \frac{1}{2(\pi a)^2} \int dx \{ g^{(2)}_{oo\pi\pi} \cos 2\theta_{\rho-} \cos 2\phi_{\sigma-} - g^{(2)}_{o\pi o\pi} \cos 2\phi_{\rho-} \cos 2\theta_{\sigma-} \tag{5}$$
$$+ g^{(4)}_{oo\pi\pi} \cos 2\phi_{\rho-} \cos 2\theta_{\rho-} - g^{(4)}_{o\pi o\pi} \cos 2\phi_{\sigma-} \cos 2\theta_{\sigma-} \},$$

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$$H_3 = \frac{1}{2(\pi a)^2} \int dx \{ g^{(1)}_{oooo} \cos 2\phi_{\sigma-} + g^{(1)}_{o\pi\pi o} \cos 2\phi_{\rho-} + g^{(1)}_{o\pi o\pi} \cos 2\theta_{\sigma-} - g^{(1)}_{oo\pi\pi} \cos 2\theta_{\rho-} \} \cos 2\phi_{\sigma+}, \quad (6)$$

where a is the lattice constant. H_2 is associated with interband processes induced by intra-chain forward scattering, and H_3 with the intrachain backward scattering. The sign of each term in Eqs.(5) and (6) is determined according to a simple algebra of Majorana Fermions [11], introduced to preserve the proper anticommutation relations between Fermion fields with differing band and spin indices. The interaction parameter $g_{\alpha\beta\gamma\delta}$ represents the scattering from (δ, γ) to (α, β). All g's in Eqs. (5) and (6) are equal to U for the case of Hubbard model. We used in Eqs. (5) and (6) that $g^{(1)}_{oooo} = g^{(1)}_{\pi\pi\pi\pi}$ and the similar properties of g's.

The impurity scatterings are described by

$$H_{imp} = \sum_{\alpha,s,i} V^{imp}_{\alpha i} n_{\alpha si}. \quad (7)$$

Applying the bosonization to H_{imp} and separate it into the forward and backward scattering parts, $H_{imp} = H_f + H_b$, we get

$$H_f = \int dx \eta_1(x)(\partial_x \phi^o_\rho + \partial_x \phi^\pi_\rho) \quad (8)$$
$$+ \int dx \frac{\eta_2}{\pi a} [e^{i(\phi_{\rho-}+\theta_{\rho-})} \cos(\phi_{\sigma-} + \theta_{\sigma-}) + e^{i(-\phi_{\rho-}+\theta_{\rho-})} \cos(\phi_{\sigma-} - \theta_{\sigma-}) + \text{h.c}],$$
$$H_b = \frac{1}{\pi\alpha} \int dx \xi_1(x) \{ e^{-i(\phi_{\rho+}+\phi_{\rho-})} \cos(\phi_{\sigma+} + \phi_{\sigma-}) + e^{-i(\phi_{\rho+}-\phi_{\rho-})} \cos(\phi_{\sigma+} - \phi_{\sigma-}) \} + \text{h.c.}$$
$$+ \frac{1}{\pi\alpha} \int dx \xi_2(x) \{ e^{-i(\phi_{\rho+}+\phi_{\sigma+})} \cos(\theta_{\rho-} + \theta_{\sigma-}) + e^{-i(\phi_{\rho+}-\phi_{\sigma+})} \cos(\theta_{\rho-} - \theta_{\sigma-}) \} + \text{h.c.} \quad (9)$$

η_1 and ξ_1 are the random potential fields within o and π bands, and η_2 and ξ_2 are the random hopping between o and π bands. In order to make a quantitative comparison with the single chain case, we use the same distribution for η and ξ as used in the single chain model. In the single chain model we assumed Gaussian distribution and $\overline{\eta(0)\eta(x)} = D\delta(x)$, $\overline{\xi^*(0)\xi(x)} = W\delta(x)$. We assume the same for $V^{imp}_{\alpha i}$, that is, we have $\overline{\eta_i(0)\eta_j(x)} = (D_i/2)\delta_{ij}\delta(x)$ and $\overline{\xi^*_i(0)\xi_j(x)} = (W_i/2)\delta_{ij}\delta(x)$, where $i, j = 1, 2$. The factor $1/2$ appeared in the transformation from $c_{1,2}$ to $a_{o,\pi}$.

The Hamiltonian H has been studied analytically [11–14] and numerically [15–19] and is shown to have a rich phase diagram. The model including impurities, that is $H + H_{imp}$, was recently investigated using a RG method [20,21]. It was found in a spin gap phase of the repulsive Hubbard ladder that the dominant coupling between charge density and impurity potentials is the $4k_F$ Fourier component, which is already shown in the single chain models to be reduced by the renormalizations. Therefore the persistent current of the spin gap phase is suppressed by the repulsive interactions, at variance with the single Hubbard chain.

The spin gap phase has attracted a lot of attentions in connection with the possibility of superconductivity. But here our final concern is to see the interplay between interactions and impurities in multi-channel systems, keeping the experimentally relevant situations in mind. Since the ground states are gapless in the two limits, a single channel and the real ring with many channels, we would like to focus on the *gapless* phase of the two-chain model [22] in order to start a systematic interpolation between the limits, although the gapless phase occupies only a small area in the phase diagram. [13]

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Here we first study the effect of H_b term. Using the replica trick for the random variables and following the standard RG procedure, we get the RG equations of the backward impurity scatterings W_1 and W_2 in the gapless phase;

$$\frac{dW_1}{dl} = [3 - (K_{\rho+} + K_{\rho-} + K_{\sigma+} + K_{\sigma-})/2 - (g_{oooo}^{(1)} + g_{o\pi\pi o}^{(1)})/2\pi u_\sigma]W_1, \tag{10}$$

$$\frac{dW_2}{dl} = [3 - (K_{\rho+} + \frac{1}{K_{\rho-}} + K_{\sigma+} + \frac{1}{K_{\sigma-}})/2 - (g_{o\pi o\pi}^{(1)} - g_{oo\pi\pi}^{(1)})/2\pi u_\sigma]W_2. \tag{11}$$

The renormalizations to $g^{(1)}$ in Eqs. (10) and (11) are described by

$$\frac{dg_{oooo}^{(1)}}{dl} = (2 - K_{\sigma+} - K_{\sigma-})g_{oooo}^{(1)} - \frac{W_1 a}{u_\sigma}, \tag{12}$$

$$\frac{dg_{o\pi\pi o}^{(1)}}{dl} = (2 - K_{\sigma+} - K_{\rho-})g_{o\pi\pi o}^{(1)} - \frac{W_1 a}{u_\sigma}, \tag{13}$$

$$\frac{dg_{o\pi o\pi}^{(1)}}{dl} = (2 - K_{\sigma+} - \frac{1}{K_{\rho-}})g_{o\pi o\pi}^{(1)} - \frac{W_2 a}{u_\sigma}, \tag{14}$$

$$\frac{dg_{oo\pi\pi}^{(1)}}{dl} = (2 - K_{\sigma+} - \frac{1}{K_{\sigma-}})g_{oo\pi\pi}^{(1)} + \frac{W_2 a}{u_\sigma}. \tag{15}$$

In the case of Hubbard model with small U and small disorder, we ignore the renormalization to K and g and then we get

$$\frac{dW_1}{dl} = [1 - U/\pi v_F]W_1, \tag{16}$$

$$\frac{dW_2}{dl} = [1 + O((U/v_F)^2)]W_2 \tag{17}$$

The coefficient of $(U/v_F)^2$ term in Eq. (17) is positive. First, we notice that Eq. (16) has the same form as Eq. (1), and the renormalization to the backward impurity scatterings within the bands is unchanged even when two chains are coupled, which is to be contrasted to the case of spin gap phase [20]. The impurity scatterings between the bands behave differently, however. The presence of the electron-electron interactions leads to the larger W_2, although the dependence of W_2 on U is rather weak since the leading order in Eq. (17) is $(U/v_F)^2$.

The current coupled to the flux penetrating the ring, is $\sum_s(j_{1s} + j_{2s}) = \sum_s(j_{os} + j_{\pi s}) = (2/\sqrt{\pi})\Pi_{\rho+}$. Therefore only $(\rho+)$ mode contributes to persistent current and charge stiffness D, where the latter is given by $4K_{\rho+}u_{\rho+}$. Then the RG equation of charge stiffness is

$$\frac{dD}{dl} = -\frac{2DK_{\rho+}u_{\rho+}a}{\pi u_{\sigma+}^3}W_+, \tag{18}$$

where $W_+ = (W_1 + W_2)/2$. Remember W_1 behaves the same way as in the single-chain case, namely U suppresses W_1, whereas W_2 increases with U. The RG equation of D for single Hubbard chain is $dD/dl = -(2DK_\rho u_\rho a/\pi u_\sigma^3)W$. One then see D of the gapless phase of two chain model *is* enhanced by the interaction U, which is in contrast to the spin gap phase, but the enhancement is weaker than single chain. Namely, the enhancement of charge stiffness (therefore persistent current) due to the electron-electron repulsive interactions becomes smaller by doubling the number of channels.

The forward scatterings from impurities, H_f, were already studied in Ref. [20,21]. The first term in H_f can be absorbed in transforming the definition of $\phi_{\rho\pm}$ and then has no effect. The second term, however, was shown to make the interaction terms effectively weak, which results in that the enhancement of the persistent current due to the presence of the interactions would become even smaller in two chains than in single chain.

IV. SUMMARY

We studied the impurity effect on the gapless phase of the two-chain Hubbard model, as a first step from a single chain analysis toward the finite cross section ring. In contrast to the spin gap phase, the RG calculations show that the charge stiffness, and hence persistent current, would be enhanced by the repulsive interactions. The enhancement, however, is smaller than in single Hubbard chain. This indicates a possibility that the large persistent current observed in the real rings with finite cross sections might not be explained by simply extending the mechanism of interaction-enhanced persistent current in single Hubbard chain. In order to draw more definite conclusion, one has to check on three chain system and work in this direction is in progress.

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